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ADVANCE CONFIDENTIAL REPORT

2.7

NOTES ON THE EFFECT OF SURFACE DISTORTIONS ON THE DRAG
AND CRITICAL MACH NUMBER OF AIRFOILS

By H. Julian Allen

Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

The effect of two-dimensional bumps and surface waviness on the pressure distribution over airfoils is considered. It is shown that the results of the analysis may be useful in evaluating the effects of accidental or intended surface distortions on the drag and critical Mach number of airfoils.

INTRODUCTION

In the development of the NACA low-drag airfoils, it was found that in order to realize extensive laminar boundary layers at high Reynolds number, the pressure gradients over the surface must be favorable (i.e., the pressure must fall progressively downstream). On commercially fabricated airplane wings, some surface waviness is generally present in the finished product. Moreover, waviness may also occur owing to the application of air loads in flight, particularly if the surface skin is thin. Waviness distorts the pressure gradients over such surfaces, so it is important to determine for low-drag wings to what extent waviness may be tolerated.

On the wings of high-speed airplanes, surface waviness - in its effect on the pressure distribution - again is of importance since it may undesirably lower the critical Mach number.

It is well known that a thin sheet of metal when subjected to a compressive load will deflect elastically in a wave shape which may be approximated by a cosine curve. In the analysis to follow, the effect of a single and a continuous two-dimensional "cosine"-wave type of surface distortion having straight-line elements in the

spanwise direction on the pressure distribution over an airfoil is considered. The results of the analysis can be used to evaluate the effect of waviness on the drag and on the critical Mach number.

ANALYSIS

Consider the two-dimensional wave distortion of the surface of an airfoil shown in figure 1. If the wave length λ of the distortion is small in comparison with the airfoil chord and the local radius of curvature, the calculation of the alteration of the airfoil velocity distribution due to the surface distribution may be satisfactorily approximated by the following procedure:

(1) The undistorted airfoil surface is considered to be replaced by a plane of infinite extent.

(2) The velocity alteration due to the wave distortion of the plane is calculated.

(3) The velocity distribution on the surface of the distorted airfoil is found by superposition of the velocity distribution over the undistorted airfoil and the velocity distribution over the distorted plane.

The Continuous Cosine Wave

The surface form of the continuous cosine wave is determined by

$$y = \frac{1}{2} h \left(1 - \cos \frac{2\pi s}{\lambda} \right); \quad -\infty < s < +\infty \quad (1)$$

where

s distance along the surface from an arbitrary reference

λ wave length

y height of the wave at the point s , as measured from the trough

and

h maximum height of the wave as measured from the trough

It is shown in reference 1 that the increment of velocity over a surface (Δv_1 at the point s_1) due to a small change in surface shape, may be determined by the approximate relation

$$\frac{\Delta v_1}{V_0} = \frac{1}{\pi} \int_{s_1}^{\infty} \frac{\frac{dy}{ds} ds}{s_1 - s} \quad (2)$$

where V_0 is the free-stream velocity, dy/ds is the change in slope of the surface at the point s , and the integration is performed for the whole interval over which the change occurs.

For the continuous cosine wave, the limits of integration are thus $-\infty$ and $+\infty$.

Differentiation of equation (1) yields

$$\frac{dy}{ds} = \pi \frac{h}{\lambda} \sin \frac{2\pi s}{\lambda}$$

By substitution, equation (2) becomes

$$\frac{\Delta v_1}{V_0} = \frac{h}{\lambda} \int_{-\infty}^{+\infty} \frac{\sin \frac{2\pi s}{\lambda} ds}{s_1 - s}$$

Again, by substitution of w for $s_1 - s$,

$$ds = -dw$$

and

$$\sin \frac{2\pi s}{\lambda} = \sin \frac{2\pi(s_1 - w)}{\lambda}$$

$$= \sin \frac{2\pi s_1}{\lambda} \cos \frac{2\pi w}{\lambda} - \cos \frac{2\pi s_1}{\lambda} \sin \frac{2\pi w}{\lambda}$$

then

$$\frac{\Delta v_1}{V_0} = -\frac{h}{\lambda} \left\{ \sin \frac{2\pi s_1}{\lambda} \int_{+\infty}^{-\infty} \frac{\cos \frac{2\pi w}{\lambda} dw}{w} - \cos \frac{2\pi s_1}{\lambda} \int_{+\infty}^{-\infty} \frac{\sin \frac{2\pi w}{\lambda} dw}{w} \right\}$$

now

$$\int_{+\infty}^{-\infty} \frac{\cos \frac{2\pi w}{\lambda} dw}{w} = 0$$

and

$$\int_{+\infty}^{-\infty} \frac{\sin \frac{2\pi w}{\lambda} dw}{w} = -\pi$$

so that the change in velocity at a point s is finally

$$\frac{\Delta v}{V_0} = -\pi \frac{h}{\lambda} \cos \frac{2\pi s}{\lambda} \quad (3)$$

The continuous cosine wave shape, and the corresponding change in the parameter $\frac{\Delta v}{V_0} \frac{\lambda}{h}$, due to the superposition on a surface of such a wave shape, are given in figure 2.

The Single Cosine Bump

For the single cosine bump,

$$y = \frac{1}{2} h \left(1 - \cos \frac{2\pi s}{\lambda} \right); \quad 0 < s < \lambda \quad (4)$$

wherein the symbols are the same as those of equation (1). The velocity change promoted by such a bump may be calculated, using the integral equation (2), provided the limits of integration are from $s = 0$ to $s = \lambda$. Substituting the slope of the bumps, obtained by differentiation of equation (4), in equation (2), gives

$$\frac{\Delta v_1}{V_0} = \frac{h}{\lambda} \int_0^\lambda \frac{\sin \frac{2\pi s}{\lambda} ds}{s_1 - s}$$

letting

$$z = \frac{2\pi}{\lambda} (s_1 - s)$$

then

$$dz = -\frac{2\pi}{\lambda} ds$$

and

$$\sin \frac{2\pi s}{\lambda} = \sin \frac{2\pi s_1}{\lambda} \cos z - \cos \frac{2\pi s_1}{\lambda} \sin z$$

so that

$$\frac{\Delta v_1}{v_0} = -\frac{h}{\lambda} \left\{ \sin \frac{2\pi s_1}{\lambda} \int_{\frac{2\pi s_1}{\lambda}}^{2\pi \left(\frac{s_1}{\lambda} - 1 \right)} \frac{\cos zdz}{z} - \cos \frac{2\pi s_1}{\lambda} \int_{\frac{2\pi s_1}{\lambda}}^{2\pi \left(\frac{s_1}{\lambda} - 1 \right)} \frac{\sin zdz}{z} \right\}$$

The definite integrals

$$\int_{\infty}^x \frac{\cos t dt}{t}$$

and

$$\int_0^x \frac{\sin t dt}{t}$$

which are functions of the upper limit x , are known as the "cosine integral" and "sine integral," respectively, and are commonly denoted by $Ci(x)$ and $Si(x)$. Tabular values of these functions are given in reference 2. The equation for the velocity increment at s is then

$$\begin{aligned} \frac{\Delta v}{v_0} = \frac{h}{\lambda} \left\{ \sin \frac{2\pi s}{\lambda} \left[Ci \left| \frac{2\pi s}{\lambda} \right| - Ci \left| 2\pi \left(\frac{s}{\lambda} - 1 \right) \right| \right] \right. \\ \left. - \cos \frac{2\pi s}{\lambda} \left[Si \left| \frac{2\pi s}{\lambda} \right| - Si \left| 2\pi \left(\frac{s}{\lambda} - 1 \right) \right| \right] \right\} \quad (5) \end{aligned}$$

provided the cosine integral is considered positive for either positive or negative values of the argument, while the sine integral is considered to have the same sign as

the argument. Values of $\frac{V}{h}$ and $\frac{\lambda}{h} \times \frac{\Delta v}{V_0}$ obtained from equations (4) and (5), respectively, are listed in table I and are shown in figure 2.

DISCUSSION AND CONCLUSIONS

The maximum velocity gradients $\frac{d(\Delta v/V_0)}{d(s/\lambda)}$ for the single cosine bump and for the continuous cosine wave may be seen from figure 2 to be about the same, the latter being slightly larger.

It follows that a single bump on the surface of a low-drag airfoil is approximately as effective in disrupting an otherwise favorable velocity gradient as is a continuous series of such bumps. As a convenient rule, the maximum velocity gradient for a single cosine bump or continuous cosine wave is $\pm 20 h/\lambda$. For a typical low-drag airfoil, such as the NACA 66,2-215 (reference 3), the

favorable gradient $\frac{d(V/V_0)}{d(s/c)}$ forward of the minimum pressure point is 0.07 at the ideal lift coefficient ($C_{L1} = 0.2$). In this case, then, a bump or continuous wave for which $h/\lambda = 0.07/20$ or 0.0035 (i.e., approx. 1/32-in. height for a 10-in. wave length) is sufficient to reduce the gradient, locally, to zero. It should be noted that, if the lift coefficient were increased to 0.4, any distortion of the upper surface forward of the minimum pressure point would promote an adverse gradient since, for $C_L = 0.4$, the gradient is zero for this portion of the undistorted airfoil.

Regarding the effects of bumps on the low-drag characteristics of such airfoils, a theoretical investigation has been made of the effect on the value of the parameter R_8^2/R_c (reference 4) of a bump of just sufficient size to reduce the favorable gradient to zero. Since this analysis showed that such bumps changed the value of R_8^2/R_c by less than 1 percent for the usual low-drag airfoils, it would be concluded that transition should be unaffected by their presence. The few available experimental observations of the influence of such bumps on transitions appear to support the results of the theoretical analysis. Whether or not surface distortions which promote unfavorable gradients are permissible is a moot question.

As regards the effect of surface distortions on the critical Mach number of airfoils, it will be noted, from figure 2, that the single bump induces a maximum velocity

$$\frac{\Delta v}{v_0} = 3.70 \frac{h}{\lambda}$$

while the continuous wave induces a maximum velocity

$$\frac{\Delta v}{v_0} = 3.14 \frac{h}{\lambda}$$

It follows that a single bump centered on the minimum pressure point is somewhat more detrimental to the critical Mach number than is the corresponding series of such bumps. Some idea of the quantitative effect of a bump on the critical Mach number can be seen from the following: The same bump ($h/\lambda = 0.0035$) which was shown to reduce the favorable gradient of the NACA 66,2-215 airfoil, at $C_L = 0.2$ to zero would, if centered on the minimum pressure point, increase the maximum value of $(v/v_0)^2$ from 1.521 (reference 3) to $[\sqrt{1.521} + 3.70 (0.0035)]^2$, which equals 1.556. This corresponds to a decrease in the critical Mach number (reference 3) from 0.693 to 0.682. Viewed in another light, this bump affects the critical Mach number as much as increasing the thickness-chord ratio of the airfoil from 0.15 to 0.16.

For distortions of short wave length for which the maximum height is of the same order or less than the boundary-layer thickness, there is some indication, both from theory and experiment, that the velocities induced by such distortions fall far short of those predicted. Therefore, for the present at least, such effects of waviness as have been shown herein must be considered to be, primarily, of qualitative rather than quantitative importance in those instances wherein the wave height is less than the boundary-layer thickness.

By combining the effects upon shape of suitably placed cosine bumps of various sizes, it is possible to simulate practically any desired change in shape, and the over-all effect of the desired change upon velocity distribution may then be determined by superposition of the effects of the individual bumps.

For example, figure 3(a) shows a rounded-edge lap joint, simulated by adding cosine bumps at half wave-length intervals downstream from an initial bump which represents the rounded edge of the lap. Figure 3(b) shows the resultant change in velocity distribution as determined by this method.

In regard to the accuracy of the principle of superposition of velocities and shapes, it should be pointed out that such superposition is satisfactory, provided the radius of curvature of the surface to which the bump is added is large. Thus, superposition on an airfoil surface will give accurate results except in the vicinity of the rounded leading edge. When it is desired to find the effect of some modification near the leading edge of an airfoil, the graphical method of reference 5 is recommended. Moreover, the single cosine bump may be satisfactorily employed on some three-dimensional bodies. It may be used, for example, with reasonable accuracy to find the change in velocity distribution over a body of revolution (e.g., a fuselage or nacelle) due to some small change in dimensions.

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4. Jacobs, Eastman N., and von Doenhoff, Albert E.: Formulas for Use in Boundary-Layer Calculations on Low-Drag Wings. NACA A.C.R., Aug. 1941.
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TABLE I.- INDUCED VELOCITY DUE TO A SINGLE COSINE BUMP

| $\frac{s}{\lambda}$ | $\frac{y}{h}$ | $\frac{\Delta v}{v_0} \times \frac{\lambda}{h}$ |
|---------------------|---------------|---|
| 0.50 | 1.000 | 3.704 |
| 0.55 or 0.45 | .976 | 3.554 |
| .60 or .40 | .906 | 3.119 |
| .65 or .35 | .794 | 2.444 |
| .70 or .30 | .655 | 1.598 |
| .75 or .25 | .500 | .670 |
| .80 or .20 | .346 | -.239 |
| .85 or .15 | .206 | -1.028 |
| .90 or .10 | .096 | -1.591 |
| .95 or .05 | .024 | -1.818 |
| 1.00 or 0 | 0 | -1.418 |
| 1.05 or -.05 | 0 | -.864 |
| 1.10 or -.10 | 0 | -.646 |
| 1.20 or -.20 | 0 | -.415 |
| 1.30 or -.30 | 0 | -.297 |
| 1.40 or -.40 | 0 | -.227 |
| 1.50 or -.50 | 0 | -.177 |
| 2.00 or -1.00 | 0 | -.074 |
| 3.00 or -2.00 | 0 | -.026 |
| 5.00 or -4.00 | 0 | -.008 |

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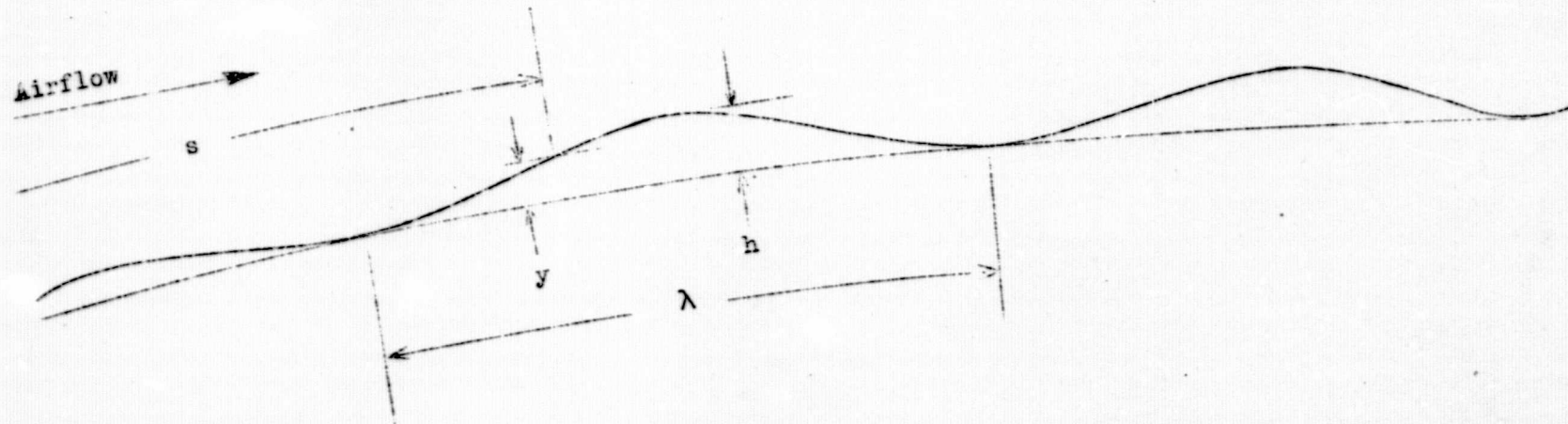


Figure 1.- Wave form on airfoil surface.

Fig. 1

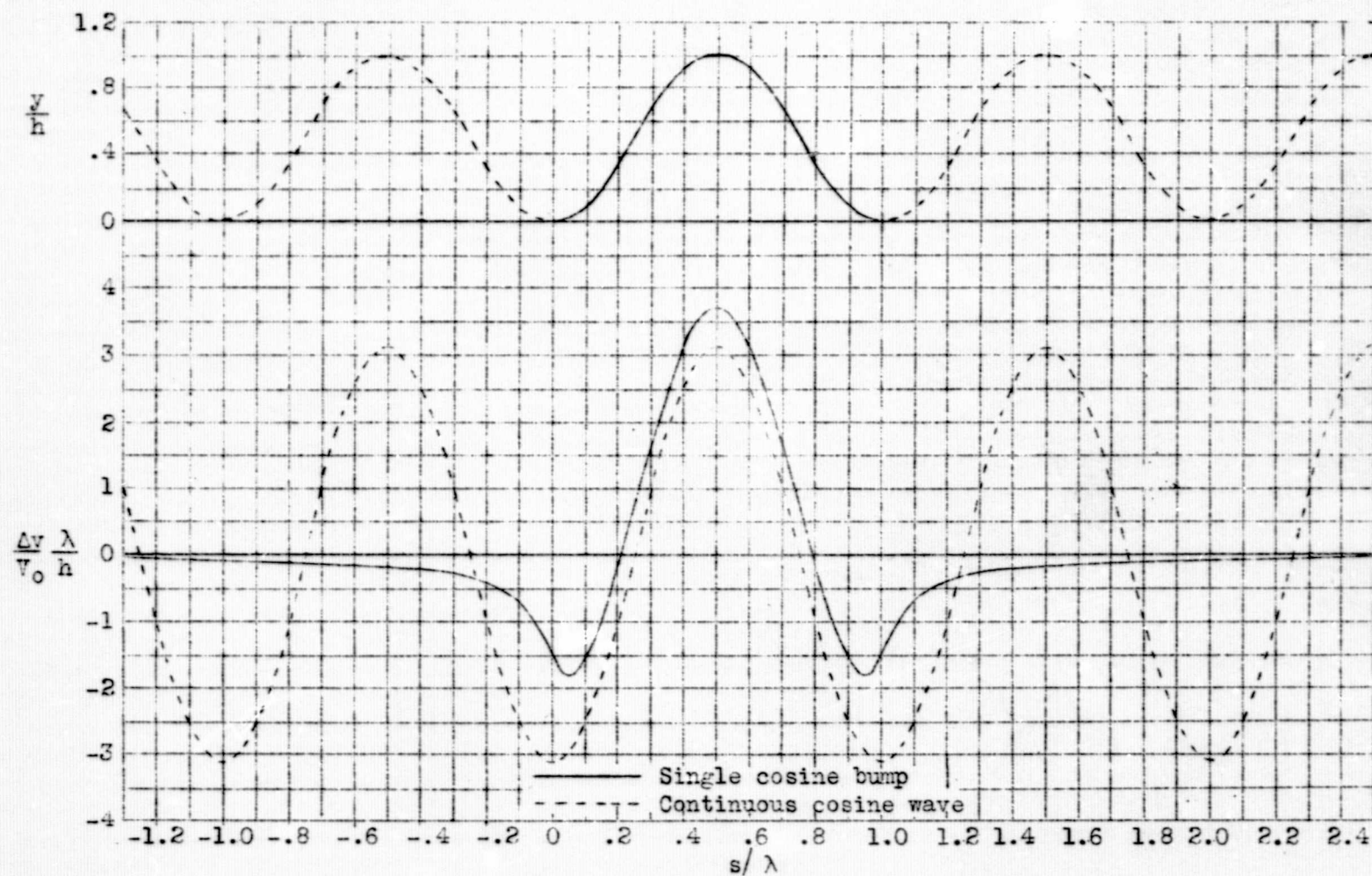


Figure 2.- Velocity distribution in the vicinity of a single cosine bump and a continuous cosine wave.

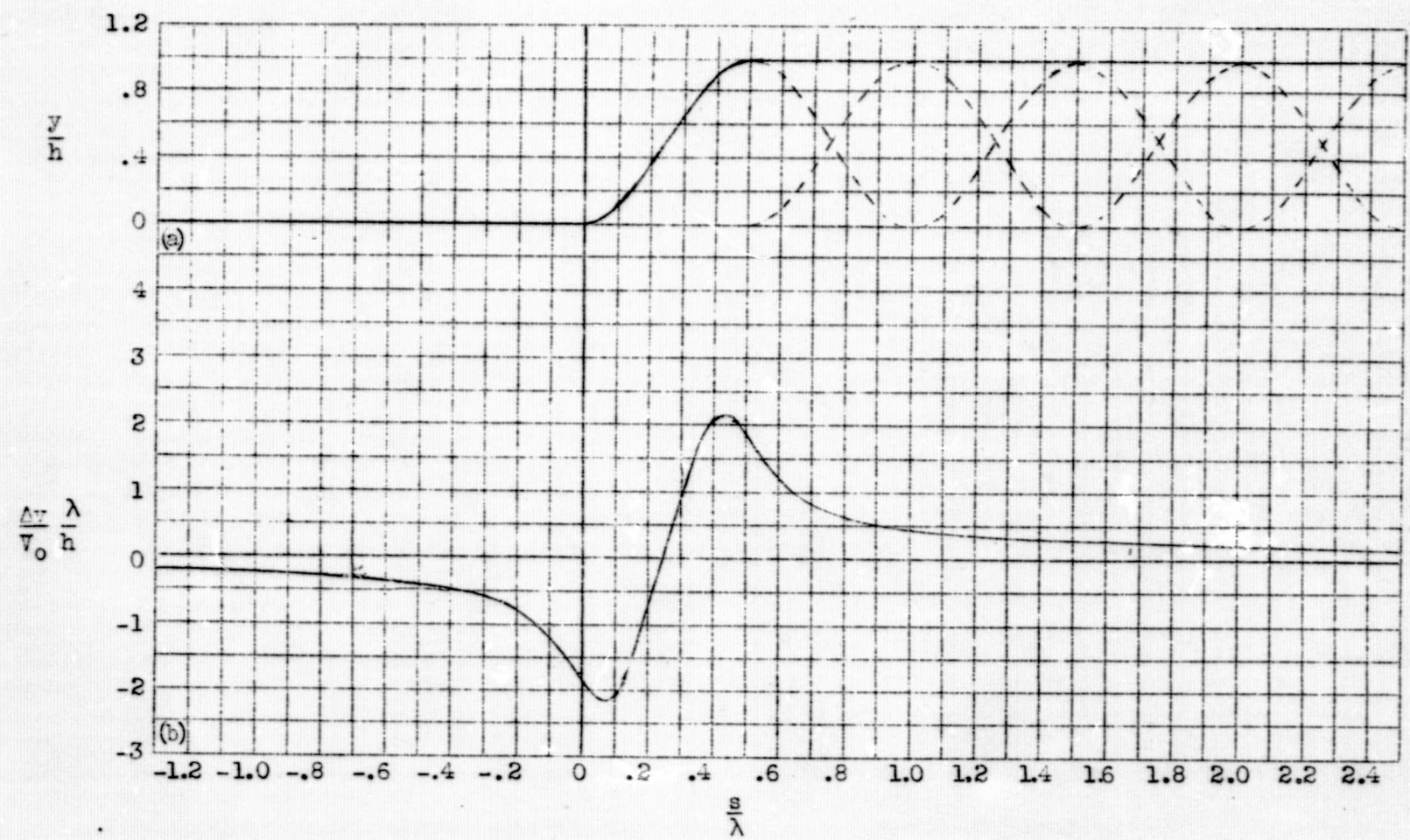


Figure 3.- Velocity distribution in the vicinity of a lap joint.